Electronics

Bode Plot

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Outline

- Bode plot introduction
- Sketch Bode plot for a given transfer function

Bode Plot

A Bode plot describes the frequency response $H(\omega)$ of a system.
Frequency response $H(\omega)$ is derived from the transfer function $H(s)$

$$H(\omega) = H(s)\Big|_{s=0+j\omega}$$

Bode plots shows $H(\omega)$ vs frequency ω

- Magnitude: $|H(\omega)|$ in dB → $20 \cdot \log_{10}|H(\omega)|$
- Phase: $\angle H(\omega)$
- ω is also in log scale

Single Pole System Bode Plot

$$H(\omega) = \frac{A}{1 - \frac{j\omega}{p_1}}$$

Stability requirement: $\mathrm{Re}\{p_1\} < 0$ (the system doesn't grow unbounded over time)
Real valued requirement: $\mathrm{Im}\{p_1\} = 0$

$$|H(\omega)| = \frac{|A|}{\sqrt{1 + \left(\frac{\omega}{p_1}\right)^2}}$$

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|H(\omega)| = 20\log_{10}|A| - 20\log_{10}\sqrt{1 + (\omega/p_1)^2}$$

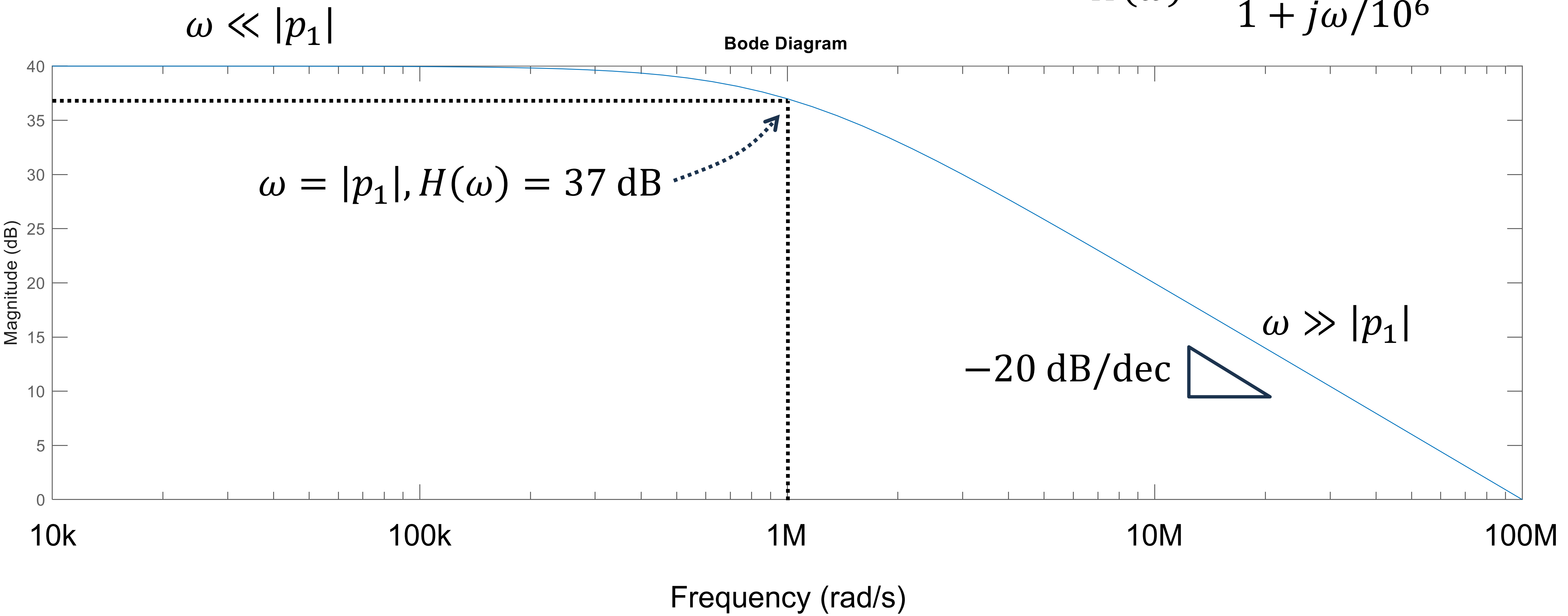
Single Pole System Bode Plot

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|H(\omega)| = 20\log_{10}|A| - 20\log_{10}\sqrt{1+(\omega/p_1)^2}$$

- $\omega \ll |p_1| \Rightarrow -20\log_{10}\sqrt{1+\left(\frac{\omega}{p_1}\right)^2} \approx -20\log_{10}\sqrt{1} = 0\ \mathrm{dB}$
  - Magnitude response is almost a flat line
- $\omega = |p_1| \Rightarrow -20\log_{10}\sqrt{1+\left(\frac{|p_1|}{p_1}\right)^2} = -20\log_{10}\sqrt{2} \approx -3\ \mathrm{dB}$
  - Magnitude response at the pole frequency is 3 dB lower that when $\omega \ll |p_1|$
- $\omega \gg |p_1| \Rightarrow -20\log_{10}\sqrt{1+\left(\frac{\omega}{p_1}\right)^2} \approx -20\log_{10}\sqrt{\left(\frac{\omega}{p_1}\right)^2} = -20\log_{10}\left|\frac{\omega}{p_1}\right|$
  - Magnitude will decrease 20 dB when ω increases by 10 times (–20 dB/decade slope)

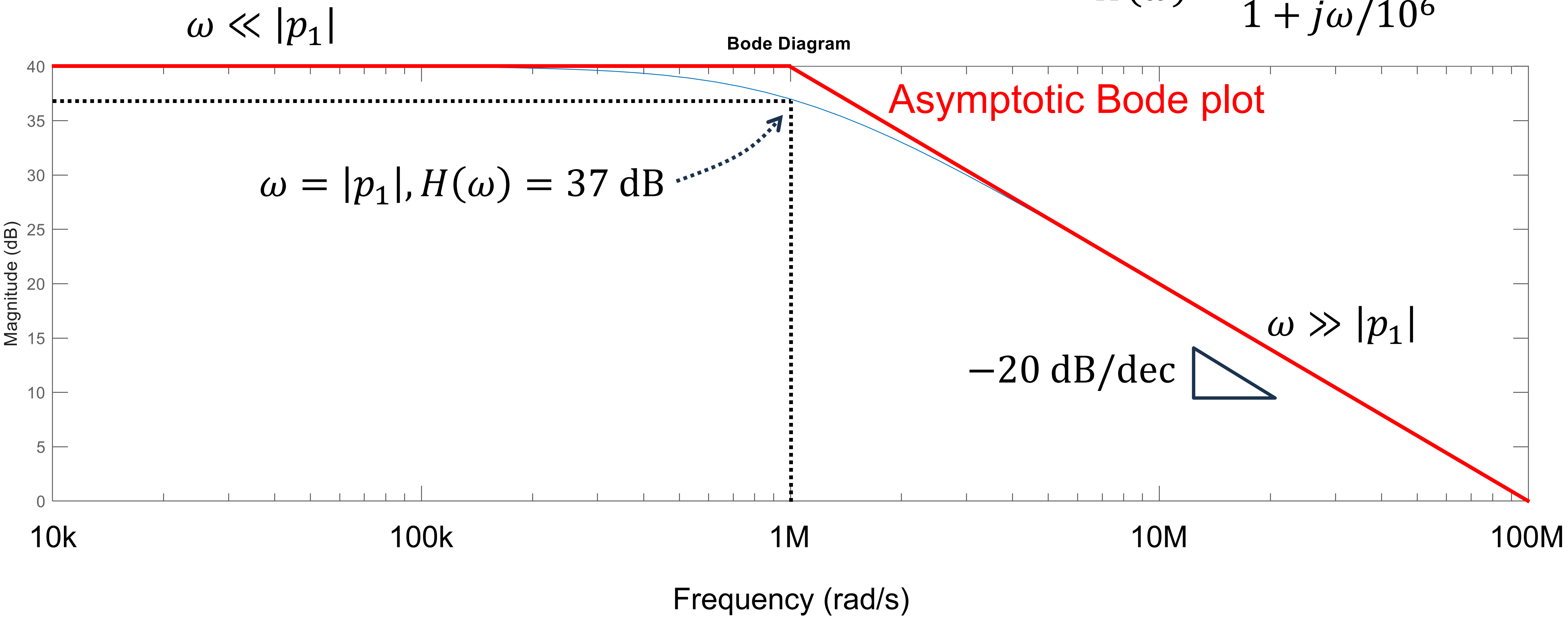
Single Pole System Bode Plot Magnitude

$$H(\omega) = \frac{100}{1 + j\omega/10^6}$$



Single Pole System Bode Plot Magnitude

$$H(\omega) = \frac{100}{1 + j\omega/10^6}$$



Single Pole System Bode Plot Phase

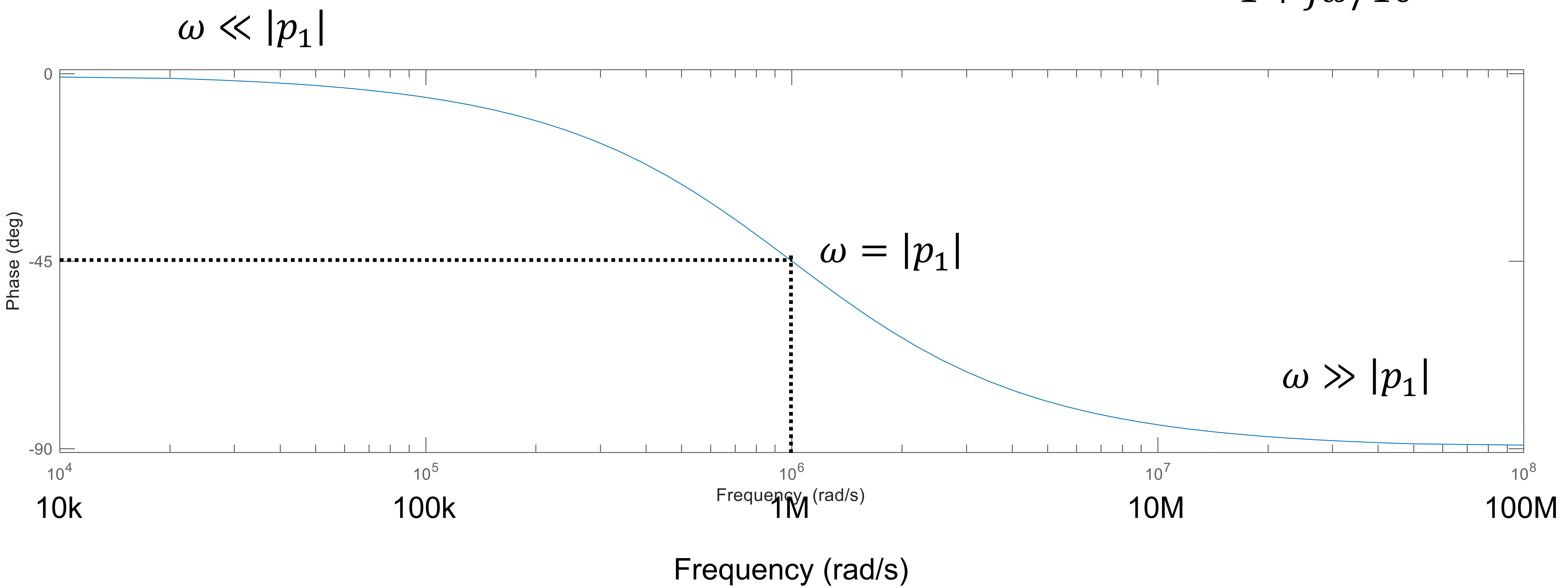
$$H(\omega) = \frac{A}{1 - \frac{j\omega}{p_1}}$$

$$\angle H(\omega) = \angle A - \angle\left(1 - \frac{j\omega}{p_1}\right) = 0 - \angle\left(1 - \frac{j\omega}{p_1}\right) = -\arctan\frac{\omega}{|p_1|}$$

- $\omega = 0 \Rightarrow \angle H(0) = -\arctan\frac{0}{|p_1|} = 0°$
- $\omega = |p_1| \Rightarrow \angle H(|p_1|) = -\arctan\frac{|p_1|}{|p_1|} = -45°$
- $\omega \gg |p_1| \Rightarrow \angle H(\omega) = \lim_{\omega\to\infty} -\arctan\frac{\omega}{|p_1|} = -90°$

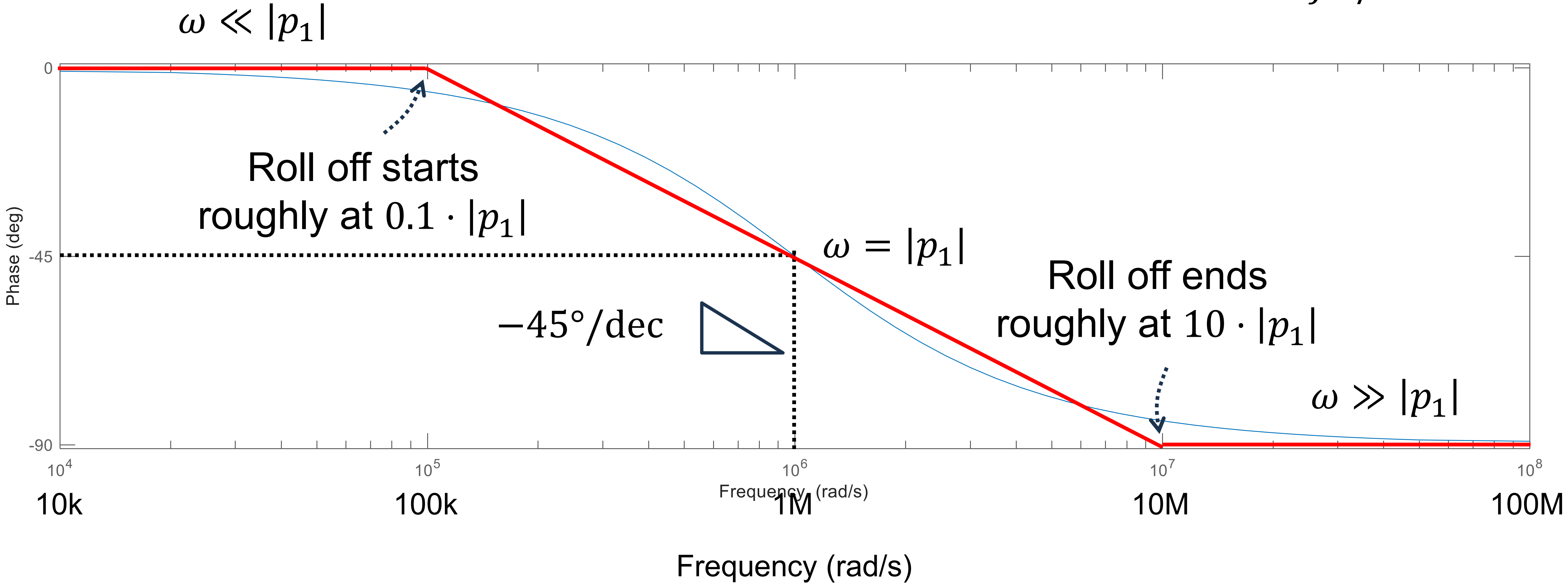
Single Pole System Bode Plot Phase

$$H(\omega) = \frac{100}{1 + j\omega/10^6}$$



Single Pole System Bode Plot Phase

$$H(\omega) = \frac{100}{1 + j\omega/10^6}$$



Single Zero System Bode Plot Magnitude

$$H(\omega) = A\left(1 - \frac{j\omega}{z_1}\right)$$

Consider: $\mathrm{Im}\{z_1\} = 0$, $\mathrm{Re}\{z_1\} < 0$ and $z_1 \neq 0$

$$|H(\omega)| = |A|\sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}$$

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|H(\omega)| = 20\log_{10}|A| + 20\log_{10}\sqrt{1 + (\omega/z_1)^2}$$

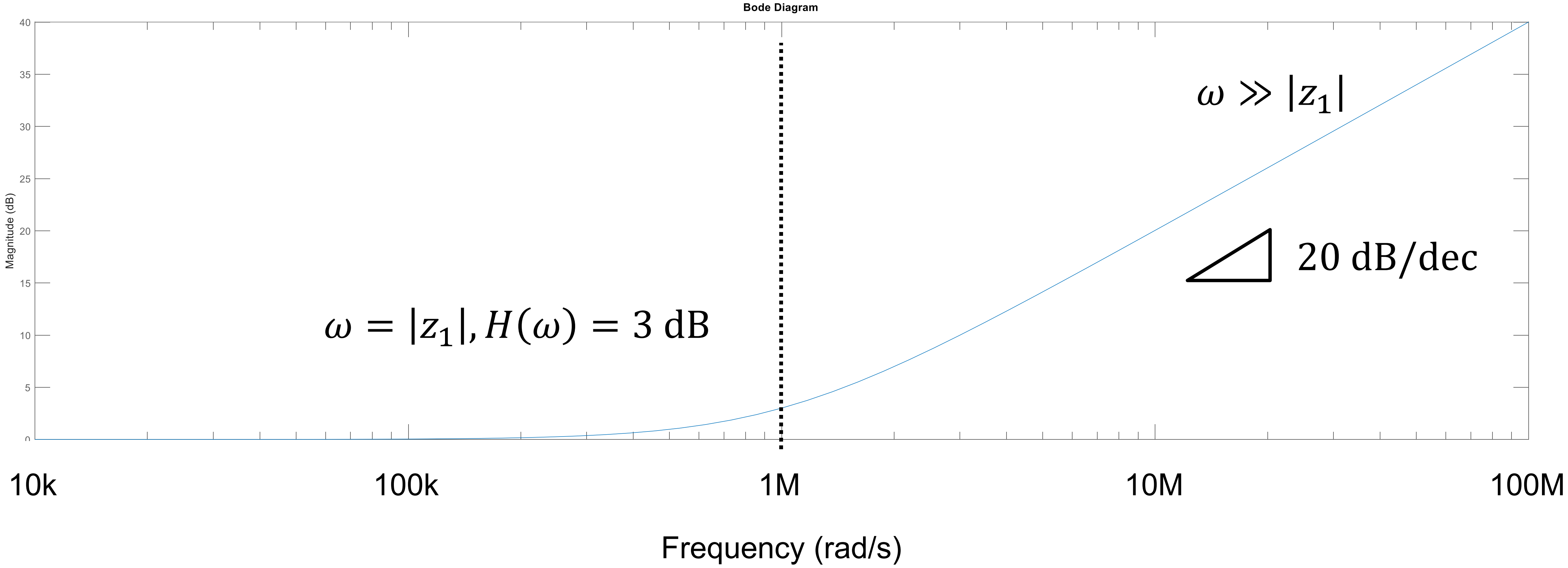
Single Zero System Bode Plot Magnitude

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|H(\omega)| = 20\log_{10}|A| + 20\log_{10}\sqrt{1+(\omega/z_1)^2}$$

- $\omega \ll |z_1| \Rightarrow 20\log_{10}\sqrt{1+\left(\frac{\omega}{z_1}\right)^2} \approx 20\log_{10}\sqrt{1} = 0\ \mathrm{dB}$
  - Magnitude response is almost a flat line
- $\omega = |z_1| \Rightarrow 20\log_{10}\sqrt{1+\left(\frac{|z_1|}{z_1}\right)^2} = 20\log_{10}\sqrt{2} \approx 3\ \mathrm{dB}$
  - Magnitude response at the zero frequency is 3 dB higher that when $\omega \ll |z_1|$
- $\omega \gg |z_1| \Rightarrow 20\log_{10}\sqrt{1+\left(\frac{\omega}{z_1}\right)^2} \approx 20\log_{10}\sqrt{\left(\frac{\omega}{z_1}\right)^2} = 20\log_{10}\left|\frac{\omega}{z_1}\right|$
  - Magnitude will increase 20 dB when ω increases by 10 times

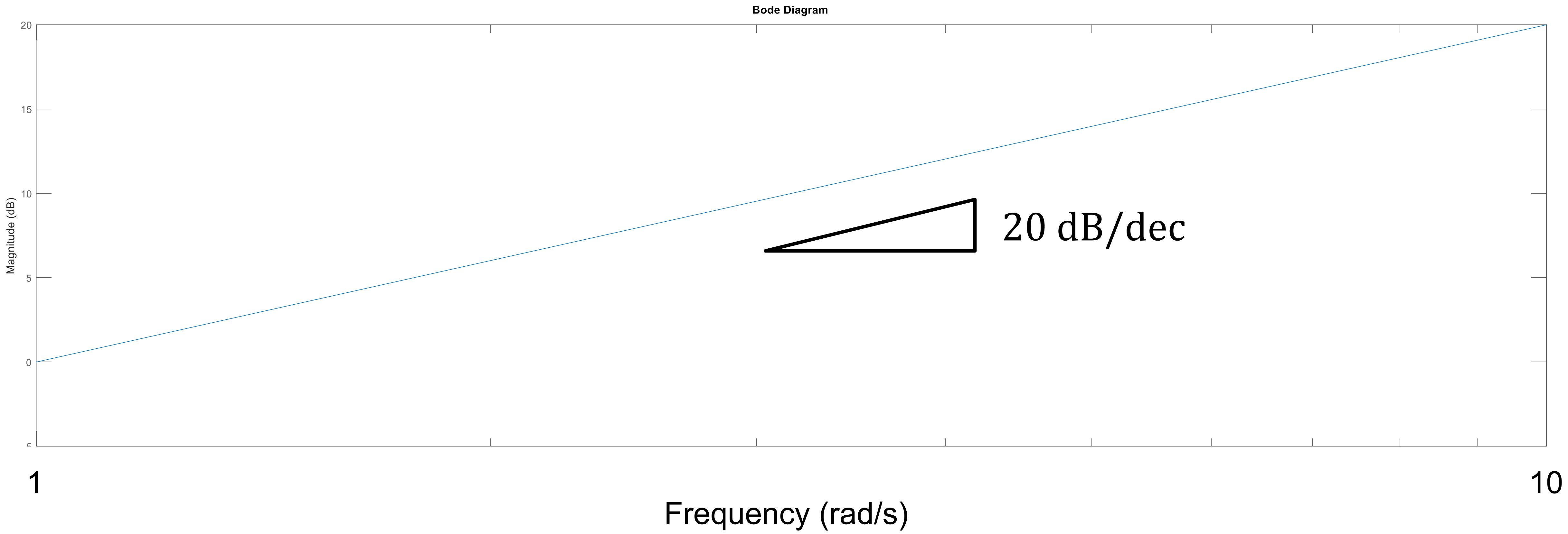
Single Zero System Bode Plot Magnitude

$$H(\omega) = 1 + j\omega/10^6$$



Single Zero System Bode Plot Magnitude

$$H(\omega) = j\omega$$
$$z_1 = 0$$



Multi-Pole System Bode Plot

$$H(s) = \frac{A}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)} \Rightarrow H(\omega) = \frac{A}{\left(1 - \frac{j\omega}{p_1}\right)\left(1 - \frac{j\omega}{p_2}\right)}$$

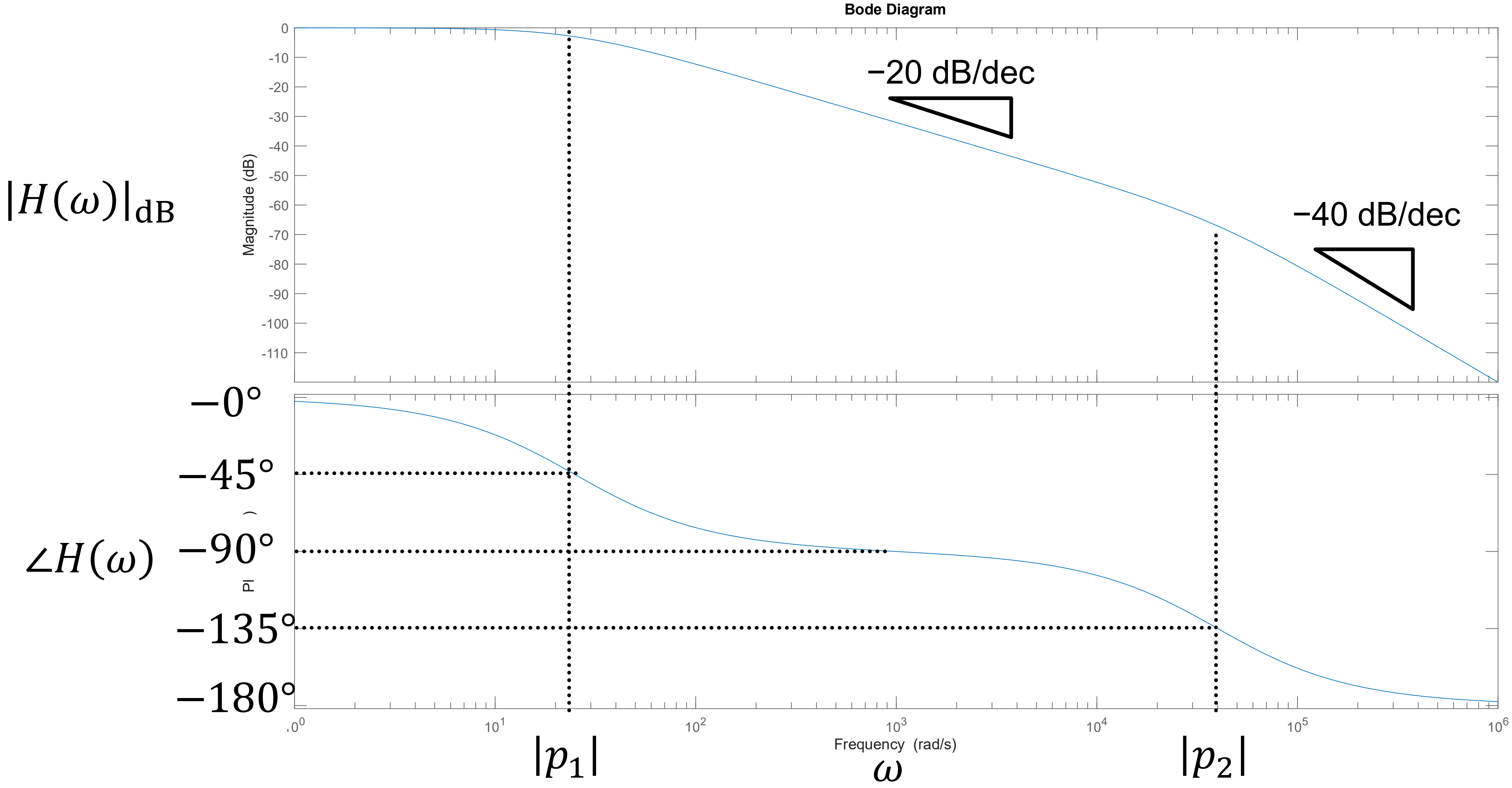
Stability requirement: $\mathrm{Re}\{p_1\}, \mathrm{Re}\{p_2\} < 0$

Assuming: $\mathrm{Im}\{p_1\} = \mathrm{Im}\{p_2\} = 0$

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|A| - 20\log_{10}\sqrt{1 + (\omega/p_1)^2} - 20\log_{10}\sqrt{1 + (\omega/p_2)^2}$$

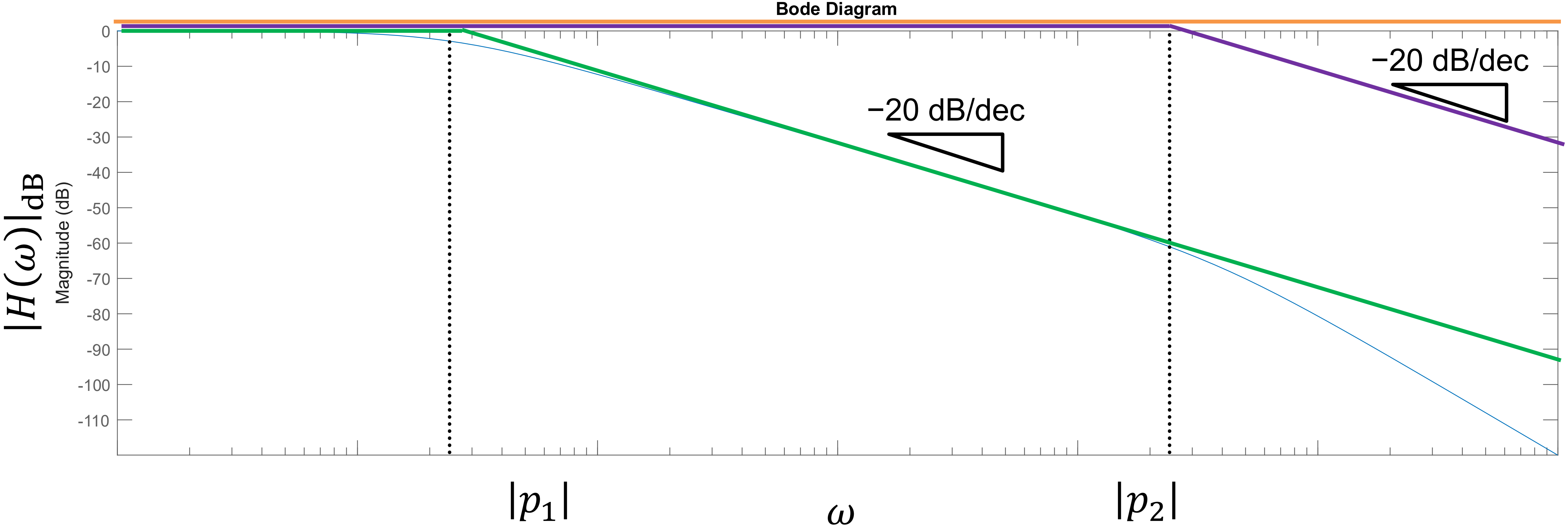
Multi-Pole System Bode Plot

$$H(s) = \frac{10^6}{s^2 + 2 \cdot 20 \cdot 10^3 s + 10^6}$$



Different Components in Bode Plot

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}|A| - 20\log_{10}\sqrt{1+(\omega/p_1)^2} - 20\log_{10}\sqrt{1+(\omega/p_2)^2}$$



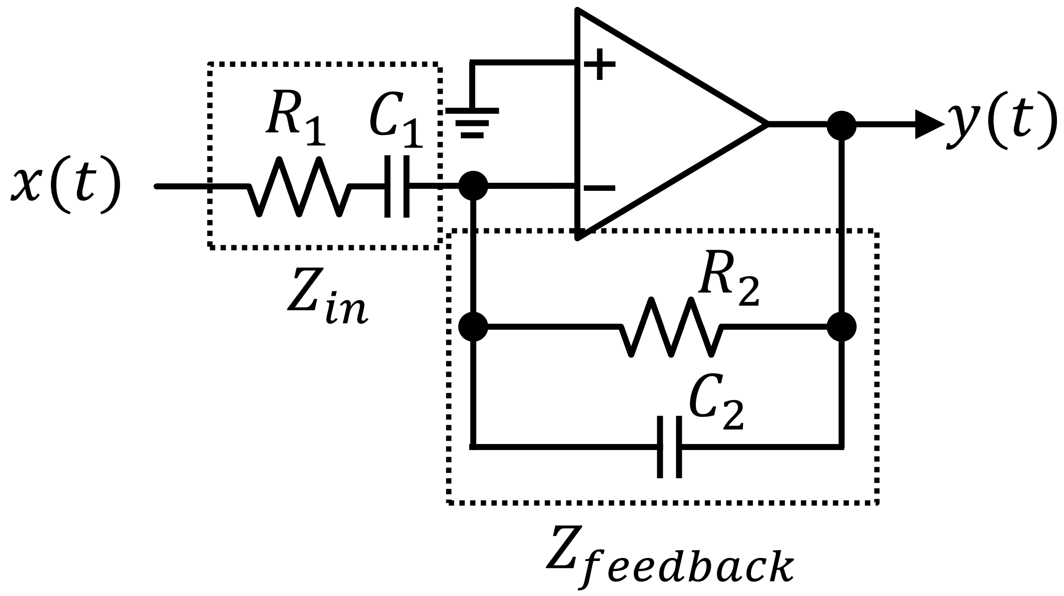
Bode Plot Procedure

- **Find $H(s)$**
  - Calculate through circuit analysis
- **Write $H(s)$ in standard form – pole-zero form**
  - $H(s) = A \cdot \frac{\left(1-\frac{s}{z_1}\right)\left(1-\frac{s}{z_2}\right)\dots\left(1-\frac{s}{z_n}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\dots\left(1-\frac{s}{p_m}\right)}$
- **Identify components**
  - $20\log_{10}|H(s)| = 20\log_{10} A + 20\log_{10}\left|1-\frac{s}{z_1}\right| + \cdots + 20\log_{10}\left|1-\frac{s}{z_n}\right| - 20\log_{10}\left|1-\frac{s}{p_1}\right| - \cdots - 20\log_{10}\left|1-\frac{s}{p_1}\right|$

Bode Plot Procedure

- Plot each component's contribution
- Add to get the total response

e.g. Practical Differentiator



Step 1: Find $\boldsymbol{H(s)}$

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{Z_{feedback}}{Zin} = -\frac{R_2 \parallel Z_{C2}}{R_1 + \frac{1}{sC_1}}$$

$$H(s) = -\frac{sR_2C_1}{(sR_2C_2 + 1)(sC_1R_1 + 1)}$$

Step 2: Write $\boldsymbol{H(s)}$ in standard form

$$H(s) = -\frac{sR_2C_1}{\left(1 - \frac{s}{-\frac{1}{R_2C_2}}\right)\left(1 - \frac{s}{-\frac{1}{C_1R_1}}\right)}$$

e.g. Practical Differentiator

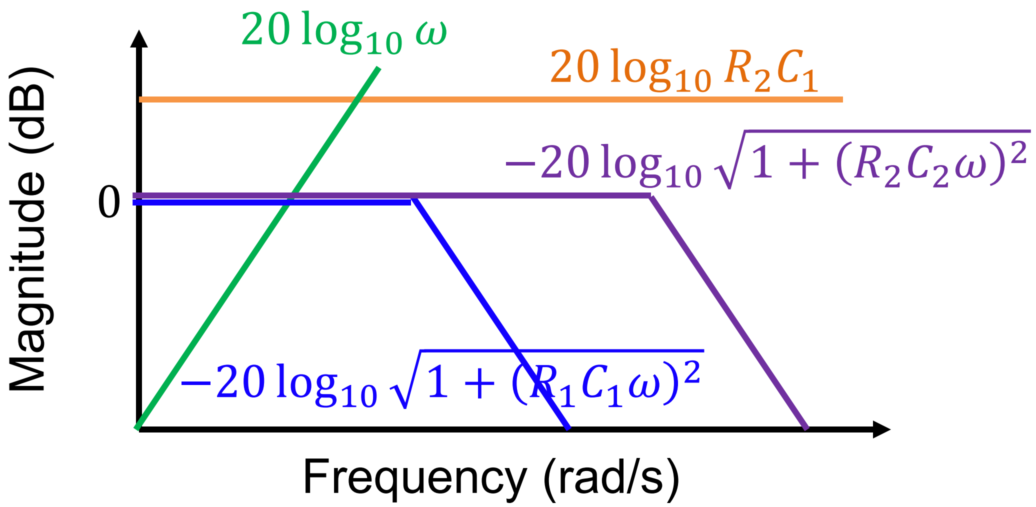
Step 3: Identify components

$$20\log_{10}|H(\omega)| \\ = 20\log_{10} R_2C_1 + 20\log_{10}\omega - 20\log_{10}\sqrt{1+(R_1C_1\omega)^2} - 20\log_{10}\sqrt{1+(R_2C_2\omega)^2}$$

Step 4: Plot each components

Step 5: Add to get total response

Step 4:



Step 5: