ECE 3250
Electromagnetic Energy Conversion
Lecture 23: Induction Machine Measurements

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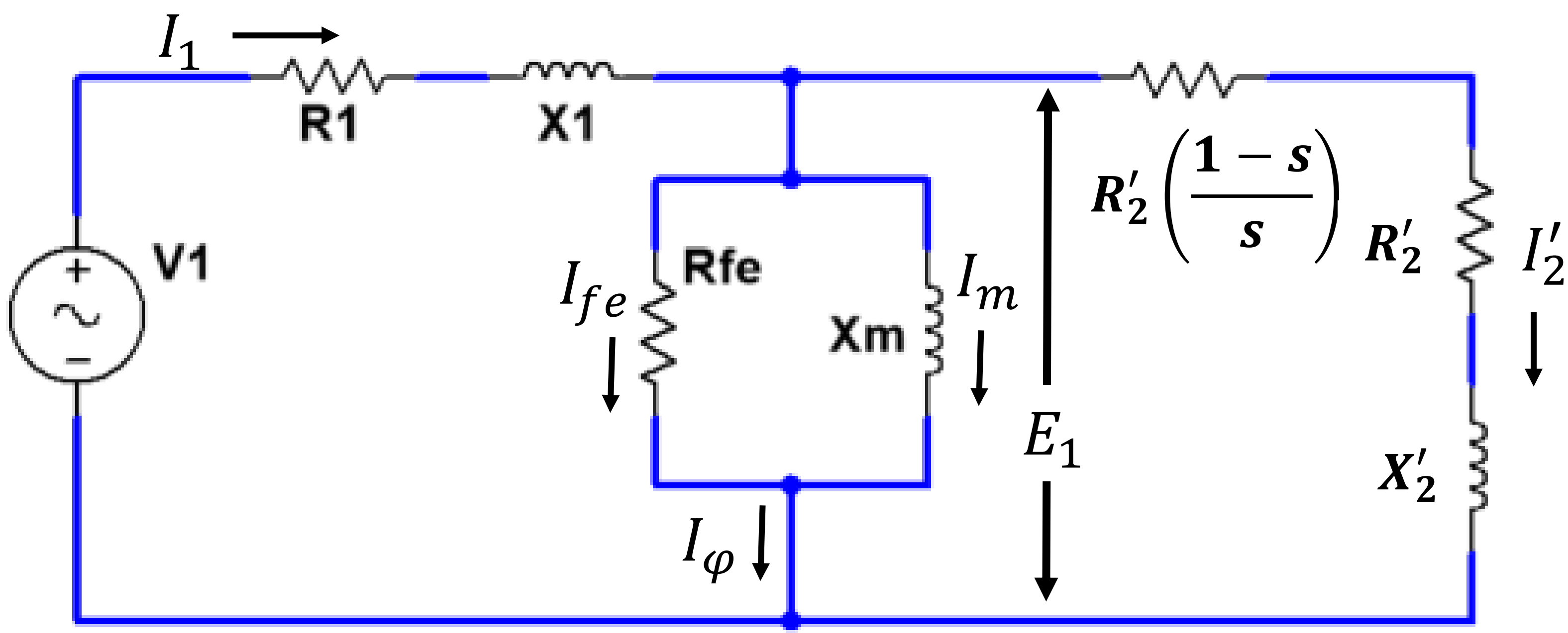
Today's Agenda

- Induction machines: taking measurements
  - No load test
  - Locked rotor test
- Induction machine efficiency

Induction Machine Circuit Model

- How do we find our induction machine circuit parameters?



Transformer vs. Motor Test

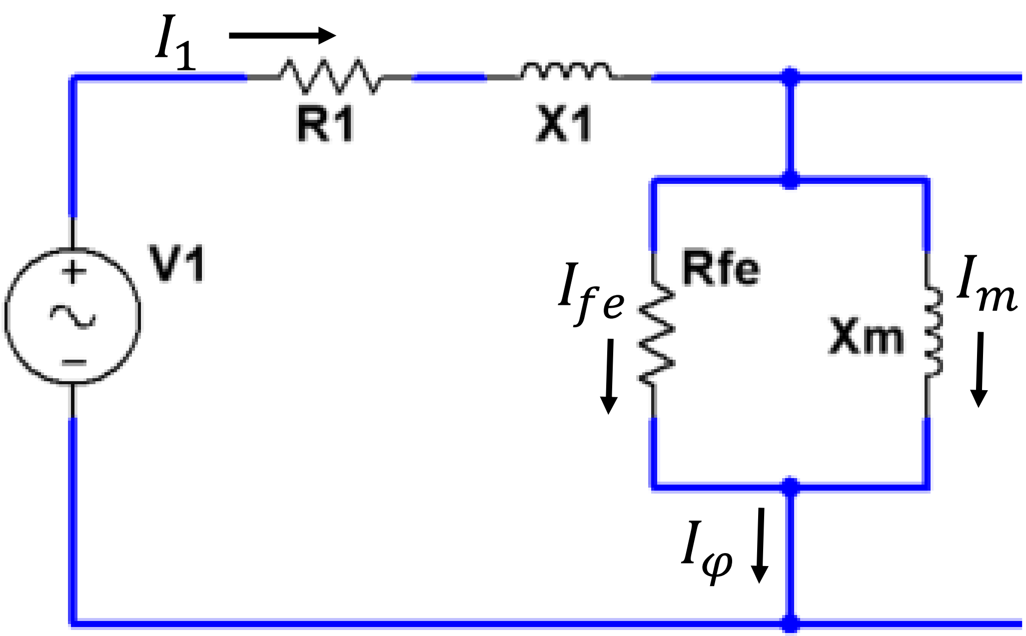
Transformer	Parameters	Motor	Parameters
Open Circuit Test	R_{fe} & jX_M	No load test (done at rated voltage)	R_{fe} & jX_M (stator)
Short Circuit Test	R_{cu} & jX_L	Locked rotor test (done at rated current)	R_2' & jX_2' (rotor)

An induction machine with no load has the rotor effectively spinning at the synchronous speed, so slip $s \approx 0$, and our $R_2'\left(\frac{1-s}{s}\right)$ term looks like an open circuit in our model

The locked-rotor induction motor is basically the same as a three-phase transformer with the stator windings acting as the primary. The rotor conductors act as short-circuited secondary windings.

No Load Test

- With no load the rotor side of the circuit is effectively removed.
- We assume that $R_1 \ll R_{fe}$ and $X_1 \ll X_m$ so losses are due almost entire to R_{fe} and X_m



No Load Test

- If there is no load on the motor, the torque is nearly zero, and the slip is also nearly zero.

$$\tau \approx 0 \qquad s \approx 0 \qquad \omega_m \approx \omega_s \qquad I_2 \approx 0$$

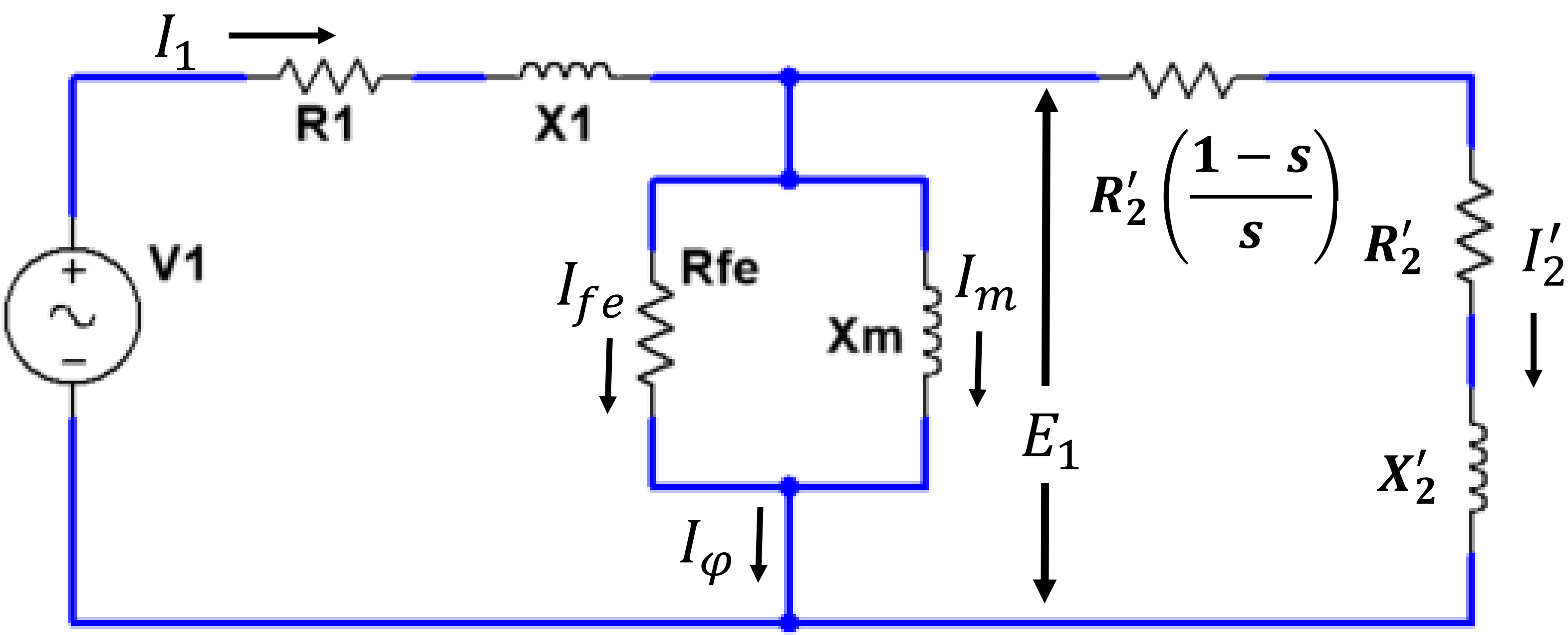
$$R_{fe} = \frac{(V_{NL})^2}{P_{NL}} \qquad Q = \sqrt{(VI)^2 - P^2} \qquad X_m = \frac{(V_{NL})^2}{Q_{NL}}$$

- Be careful to keep line and phase values straight! Since the motor is wye connected:

$$V_{phase} = \frac{V_{line}}{\sqrt{3}}$$

Locked Rotor Test

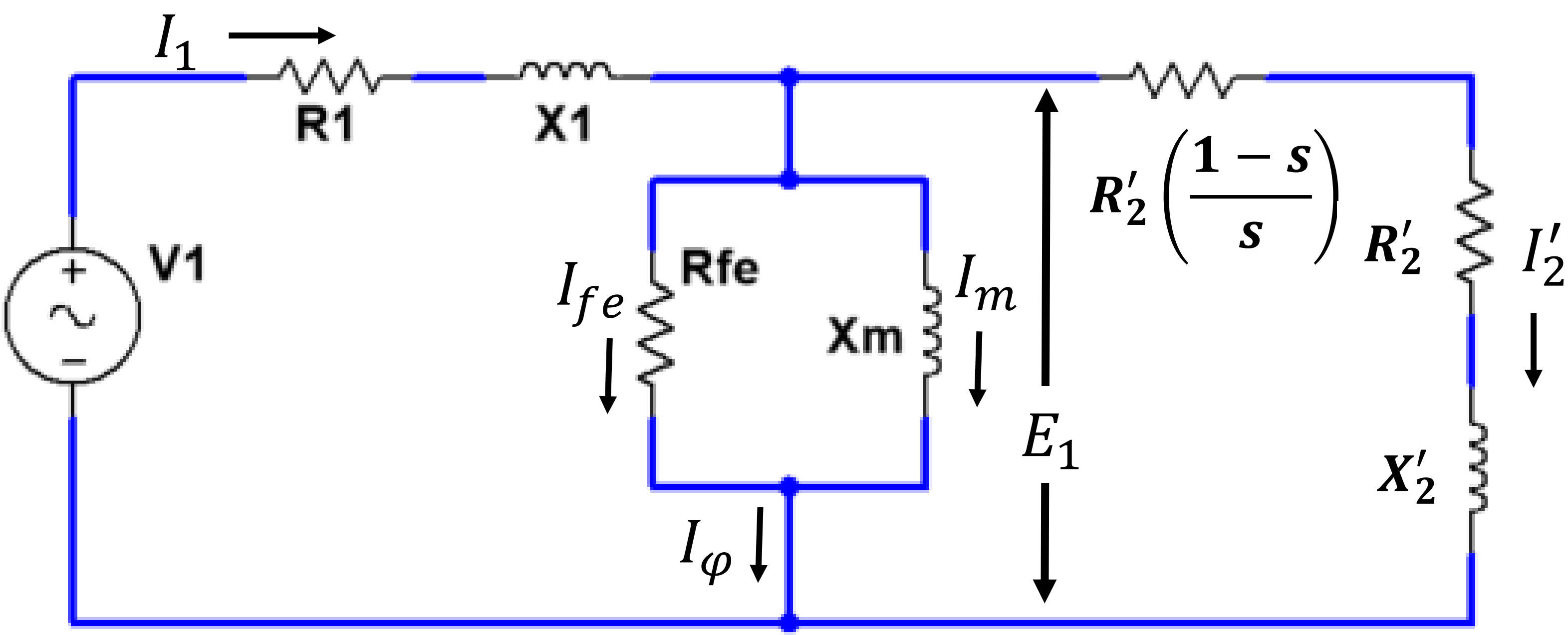
- With the rotor locked, slip = 1, so $R_2'\left(\frac{1-s}{s}\right) = 0$
- We assume $R_2' \ll R_{fe}$ and $X_2' \ll X_m$, so $I_1 \approx I_2'$ and there is almost no current in the core, so the core doesn't contribute to any losses



Locked Rotor Test

$$R_{eq} = R_1 + R_2' = \frac{P_{LR}}{(I_{LR})^2}$$

$$X_{eq} = X_1 + X_2'$$

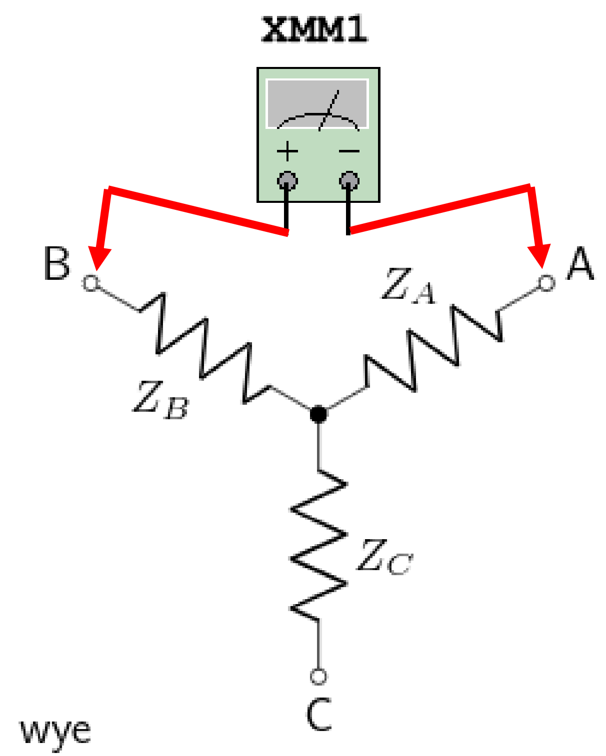


Measuring Motor Parameters

- Nearly all small motors are wye-connected and $Z_A = Z_B = Z_C$
- We can measure the resistance of the stator, R_1, with an ohm-meter, which measures only the resistance, not the inductance

$$R_1 = \frac{R_A + R_B}{2} = \frac{2R_A}{2} = R_A$$

$$R_2' = R_{eq} - R_1$$



Locked Rotor Test

- To find the reactance, we go back to Ohm's Law

$$|Z_{eq}| = \frac{V_{LR}}{I_{LR}} \qquad X_{eq} = \sqrt{(Z_{eq})^2 - (R_{eq})^2}$$

$$X_{eq} = X_1 + X_2'$$

- Here we follow an approximation that $X_1 = X_2' = \frac{X_{eq}}{2}$

Induction Motor Example

- Given a motor with the following ratings, determine test values and find the motor parameters:
- Rated Voltage: 208 V
- Rated Current: 2.77 A
- Rated Speed: 1710 rpm
- 4 pole
- Wye connected

	No Load Test (Rated Voltage)	Locked Rotor Test (Rated Current)
V	208 V	27 V
I	1.562 A	2.77 A
P	450 W	59.4 W

Induction Motor Example

- From the No Load Test

$$V_1 = \frac{V_{line}}{\sqrt{3}} = \frac{208\text{V}}{\sqrt{3}} = 120\text{V}$$

$$P_{phase} = \frac{P_{3p}}{3} = \frac{450\text{W}}{3} = 150\text{W}$$

	No Load Test (Rated Voltage)	Locked Rotor Test (Rated Current)
V	208 V	27 V
I	1.562 A	2.77 A
P	450 W	59.4 W

Induction Motor Example

- From the No Load Test

$V_1 = 120\text{V}$ $\quad\quad P_{phase} = 150\text{W}$

$$R_{fe} = \frac{(V_{NL})^2}{P_{NL}} = \frac{(120\text{V})^2}{150\text{W}} = 96\Omega$$

$$Q = \sqrt{(VI)^2 - P^2} = \sqrt{(120\text{V} \cdot 1.562\text{A})^2 - (150\text{W})^2} = 112\text{VAR}$$

$$jX_m = j\frac{(V_{NL})^2}{Q_{NL}} = j\frac{(120\text{V})^2}{112\text{VAR}} = j128\Omega$$

Induction Motor Example

- From the Locked Rotor Test

$$V_1 = \frac{V_{line}}{\sqrt{3}} = \frac{27\text{V}}{\sqrt{3}} = 15.58\text{V}$$

$$P_{phase} = \frac{P_{3p}}{3} = \frac{59.4\text{W}}{3} = 19.8\text{W}$$

$$R_{eq} = \frac{P_{LR}}{(I_{LR})^2}$$

$$= \frac{19.8\text{W}}{(2.77\text{A})^2}$$

$$= 2.58\Omega$$

	No Load Test (Rated Voltage)	Locked Rotor Test (Rated Current)
V	208 V	27 V
I	1.562 A	2.77 A
P	450 W	59.4 W

Induction Motor Example

- From direct measurement of the line-to-line resistance we find:

$$R_{line} = 2.4\Omega \quad R_1 = \frac{R_{line}}{2} = 1.2\Omega$$

$$R_2' = R_{eq} - R_1$$

$$R_2' = 2.58\Omega - 1.2\Omega = 1.38\Omega$$

Induction Motor Example

- To find the reactive components we continue using the Locked Rotor Test values

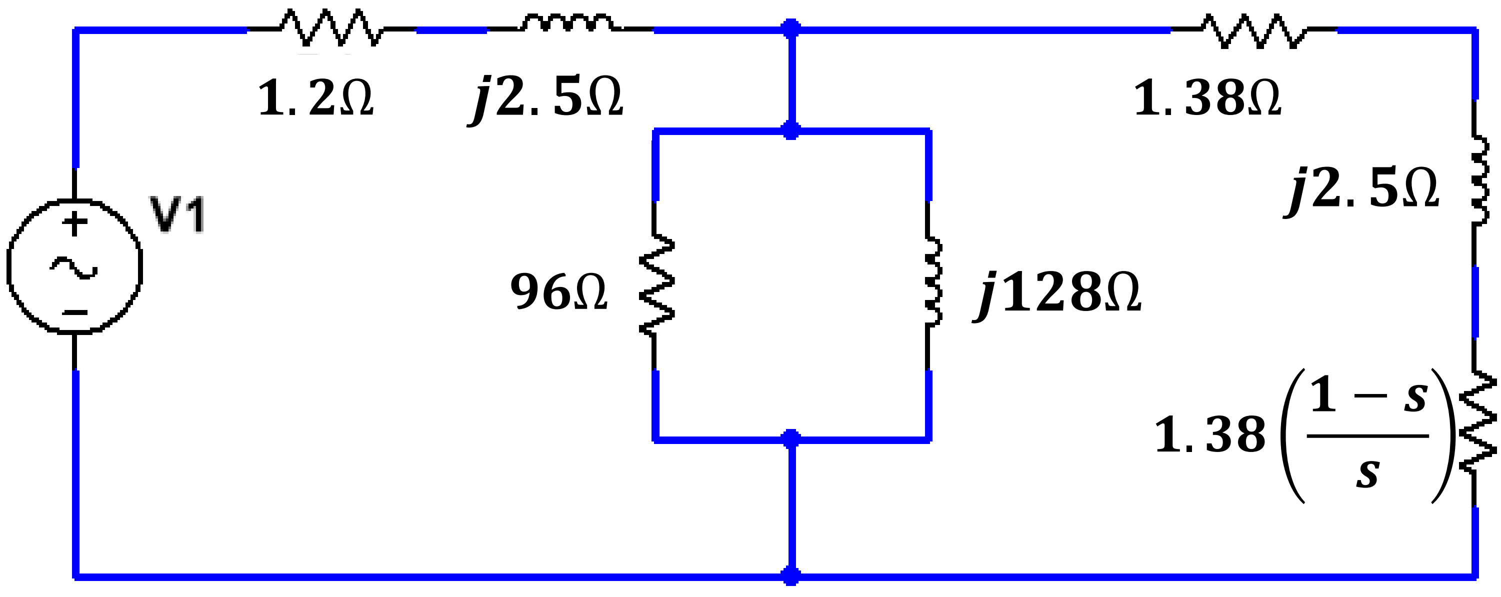
$$|Z_{eq}| = \frac{V_{LR}}{I_{LR}} = \frac{15.58\text{V}}{2.77\text{A}} = 5.63\Omega$$

$$X_{eq} = \sqrt{\left(Z_{eq}\right)^2 - \left(R_{eq}\right)^2} = \sqrt{(5.63\Omega)^2 - (2.58\Omega)^2} = 5.0\Omega$$

$$X_{eq} = X_1 + X_2' \qquad X_1 = X_2' = \frac{X_{eq}}{2} = 2.5\Omega$$

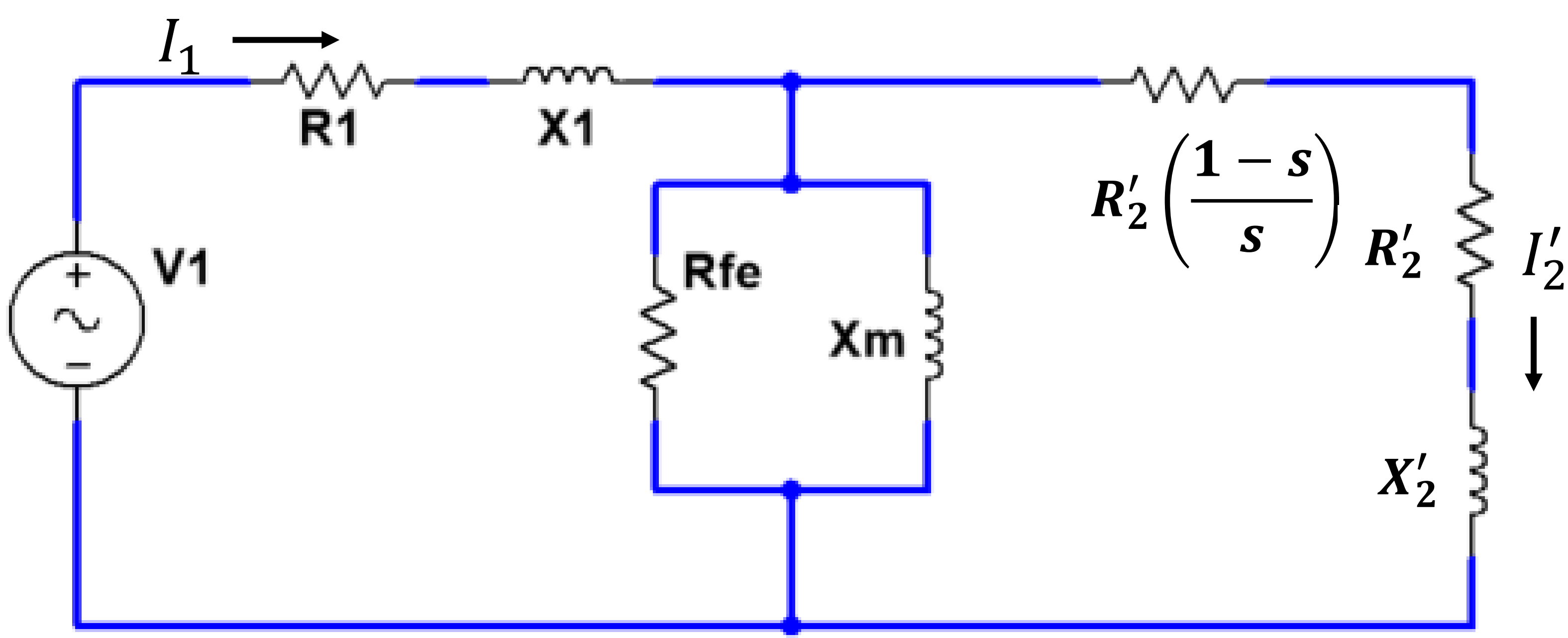
Induction Motor Example

$$\tau = \frac{P_{mech}}{\omega_m} \qquad\qquad P_{mech} = \tau\omega_m \propto 1.38\left(\frac{1-s}{s}\right)$$



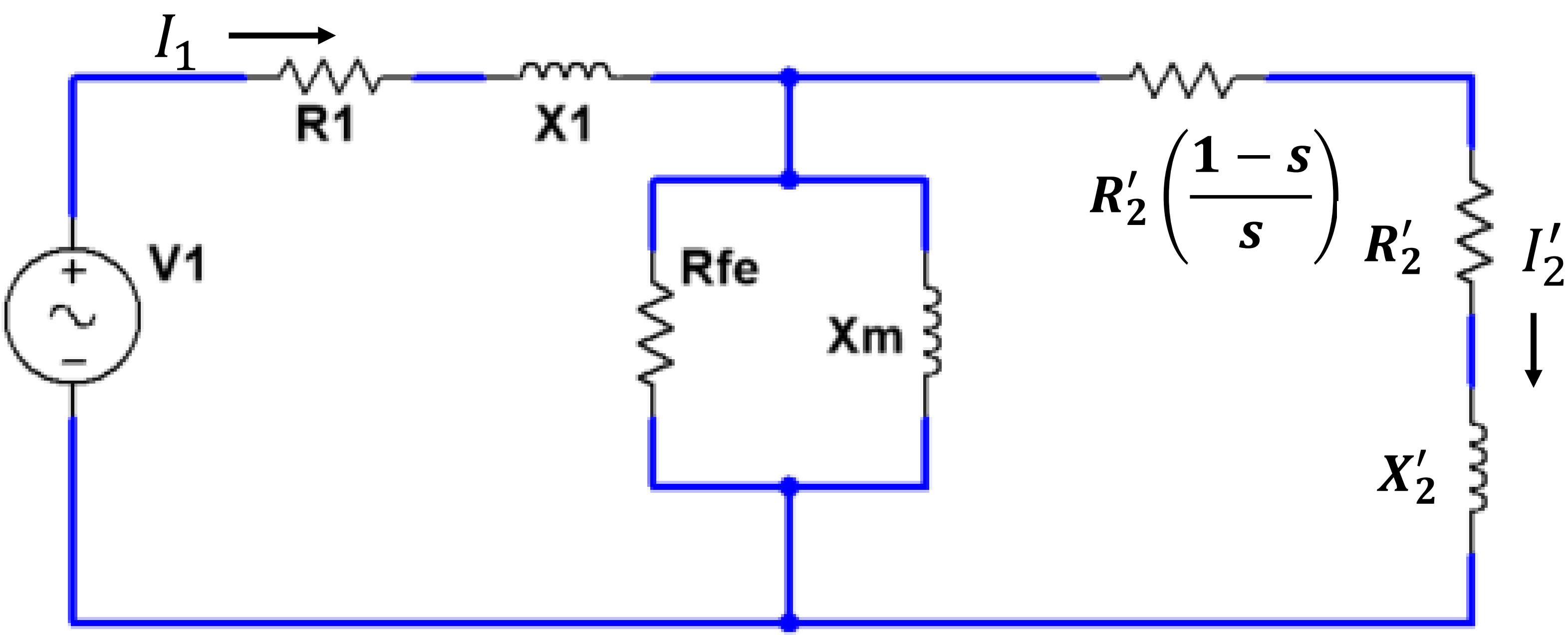
Induction Motor Efficiency & Slip

- Typically motor operates at highest efficiency at rated slip. This depends on armature winding and rotor construction.
- To get a better idea of how they are related, we return to our motor model.



Induction Motor Efficiency & Slip

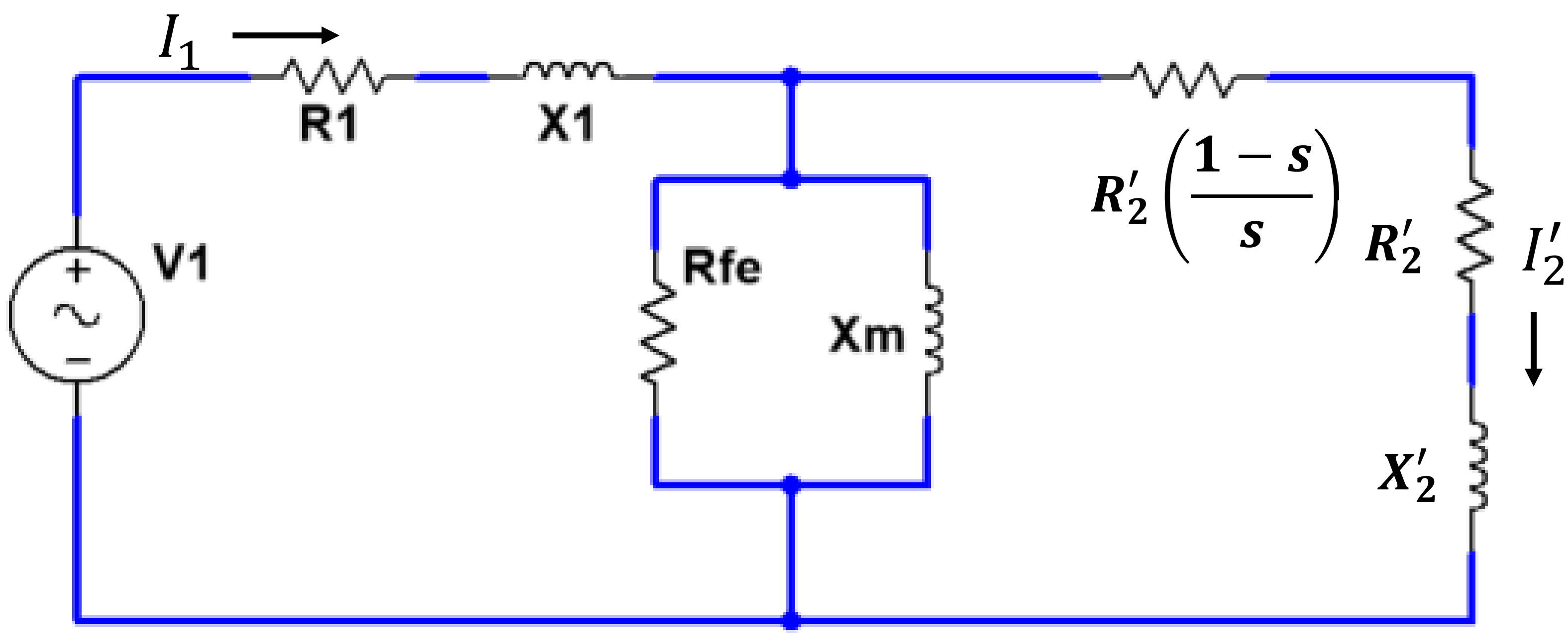
- We assume $R_2' \ll R_{fe}$ and $X_2' \ll X_m$, so we can neglect the core impedance and $I_1 \approx I_2'$
- We can then define $R_{eq} = R_1 + R_2' \qquad X_{eq} = X_1 + X_2'$



Induction Motor Efficiency & Slip

- The mechanical power developed for all 3 phases is

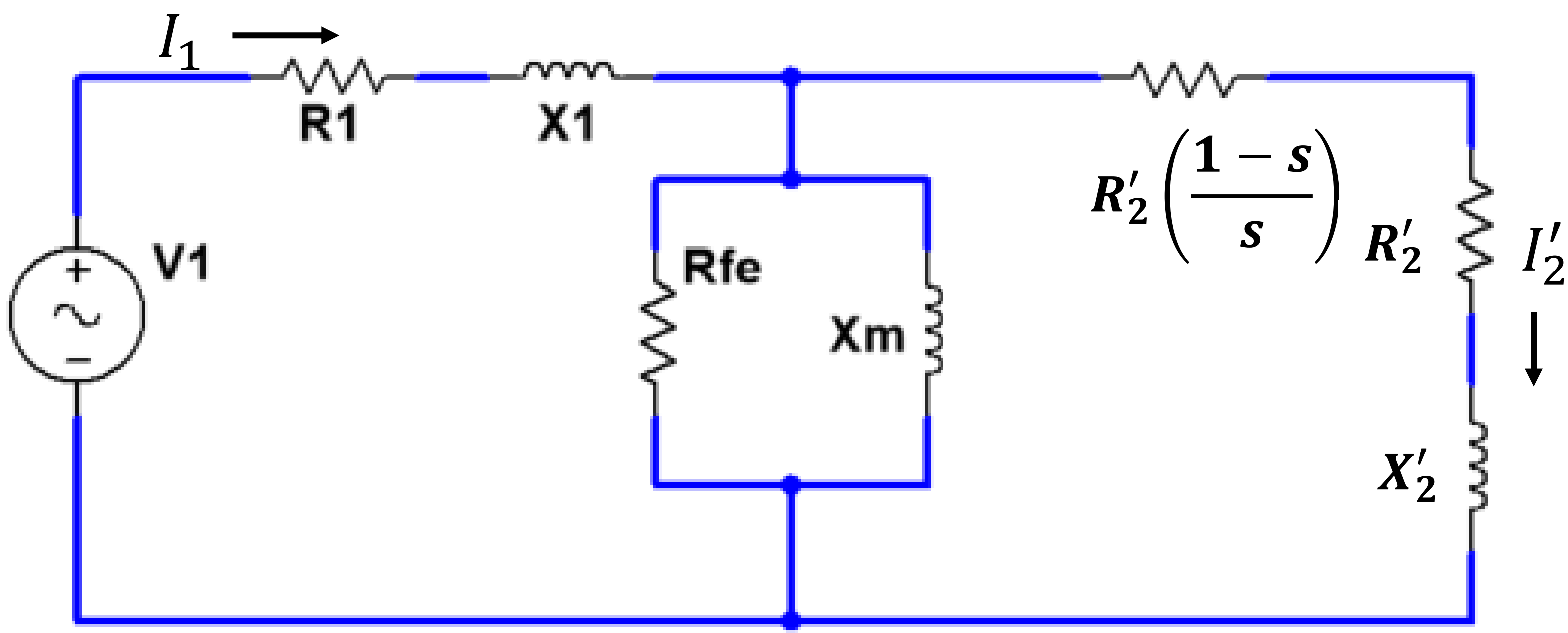
$$P_{mech} = 3(I_2')^2 R_2' \left(\frac{1-s}{s}\right)$$



Induction Motor Efficiency & Slip

- The current in terms of voltage is

$$I_2' = \frac{V_1}{\left|R_{eq} + jX_{eq} + R_2'\left(\frac{1-s}{s}\right)\right|} = \frac{V_1}{\sqrt{R_{eq}^2 + X_{eq}^2 + \left[R_2'\left(\frac{1-s}{s}\right)\right]^2 + 2R_{eq}R_2'\left(\frac{1-s}{s}\right)}}$$



Induction Motor Power & Slip

- We can now write the developed mechanical power as

$$P_{mech} = \frac{3(V_1)^2 R_2'\left(\frac{1-s}{s}\right)}{R_{eq}^2 + X_{eq}^2 + \left[R_2'\left(\frac{1-s}{s}\right)\right]^2 + 2R_{eq}R_2'\left(\frac{1-s}{s}\right)}$$

- If we want to find the slip at which we have maximum power, we can take the derivative and set it to zero. This yields:

$$R_{eq}^2 + X_{eq}^2 = \left[R_2'\left(\frac{1-s}{s}\right)\right]^2$$

Induction Motor Power & Slip

- In terms of impedance,

$$Z_{eq} = R_{eq} + jX_{eq}$$

- Then

$$Z_{eq}^2 = R_{eq}^2 + X_{eq}^2 = \left[R_2'\left(\frac{1-s}{s}\right)\right]^2$$

$$Z_{eq} = R_2'\left(\frac{1-s}{s}\right)$$

- Putting this in terms of slip: $$s_{Pmax} = \frac{R_2'}{R_2' + Z_{eq}}$$

Induction Motor Power & Slip

- Returning to our power equation,

$$P_{mech} = \frac{3(V_1)^2 R_2'\left(\frac{1-s}{s}\right)}{R_{eq}^2 + X_{eq}^2 + \left[R_2'\left(\frac{1-s}{s}\right)\right]^2 + 2R_{eq}R_2'\left(\frac{1-s}{s}\right)}$$

- Then substituting in

$$s_{Pmax} = \frac{R_2'}{R_2' + Z_{eq}}$$

- Yields and expression for maximum power:

$$P_{max} = \frac{3}{2}\left(\frac{(V_1)^2}{R_{eq} + Z_{eq}}\right)$$

Induction Motor Torque & Slip

- Recall our equation for torque $\tau = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{s\omega_s}$

$$\tau = \frac{3(V_1)^2 \frac{R_2'}{s}}{\omega_s \left[R_{eq}^2 + X_{eq}^2 + \left[R_2'\left(\frac{1-s}{s}\right)\right]^2 + 2R_{eq}R_2'\left(\frac{1-s}{s}\right)\right]}$$

- If we want to find the slip at which we have maximum torque (that is the break-down torque), we can take the derivative and set it to zero. This yields:

$$s_{\tau max} = s_B = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$

Induction Motor Torque & Slip

- We can then write the breakdown torque as

$$\tau_B = \tau_{max} = \frac{3(V_1)^2}{2\omega_s}\left[\frac{1}{R_1 + \sqrt{R_1^2 + X_{eq}^2}}\right]$$

- Remember to keep units straight! Torque is in Newton-meters and power is in Watts.

Induction Motor Efficiency & Slip

- Since the developed mechanical power varies with slip, we would expect the efficiency, η, to vary as well.

$$\eta = \frac{P_{mech}}{P_{in}}$$

Mechanical power term

Wire losses

Core losses

$$P_{in} = \frac{3(V_1)^2\left[R_2'\left(\frac{1-s}{s}\right) + R_{eq}\right]}{R_{eq}^2 + X_{eq}^2 + \left[R_2'\left(\frac{1-s}{s}\right)\right]^2 + 2R_{eq}R_2'\left(\frac{1-s}{s}\right)} + \frac{3(V_1)^2}{R_{fe}}$$

Induction Motor Example

- Given a motor with the following ratings, determine test values and find the motor parameters:
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- Rated Current: 2.77 A
- Rated Speed: 1710 rpm
- 4 pole
- Wye connected

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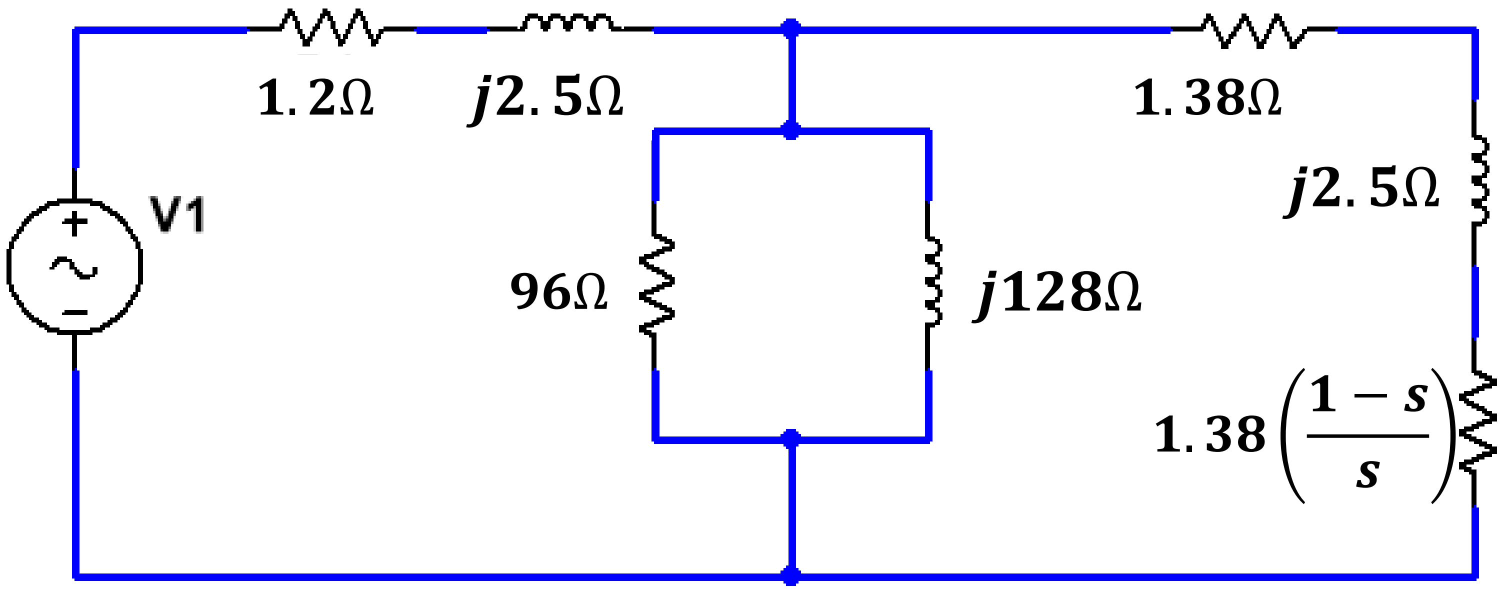
Induction Motor Example

Rated slip is given by: $s = \dfrac{n_s - n_m}{n_s} = \dfrac{1800 - 1710}{1800} = 0.05$



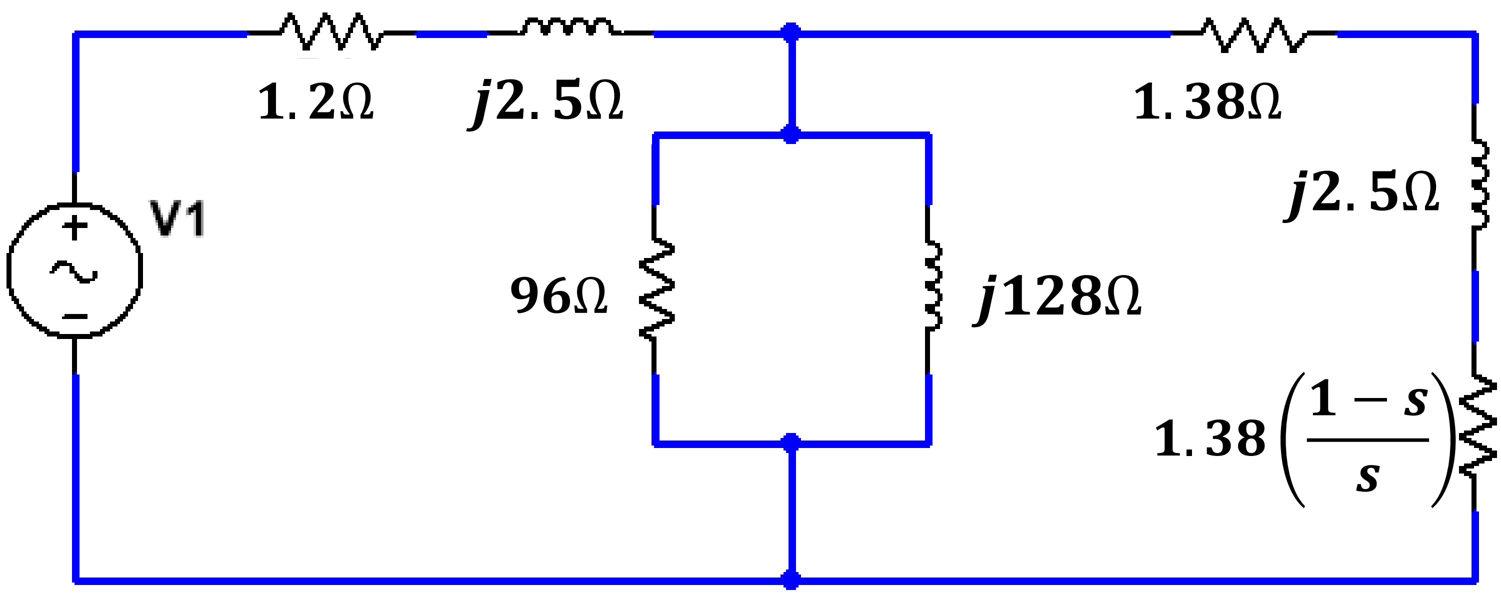
Induction Motor Example

Synchronous frequency for a 4-pole machine:

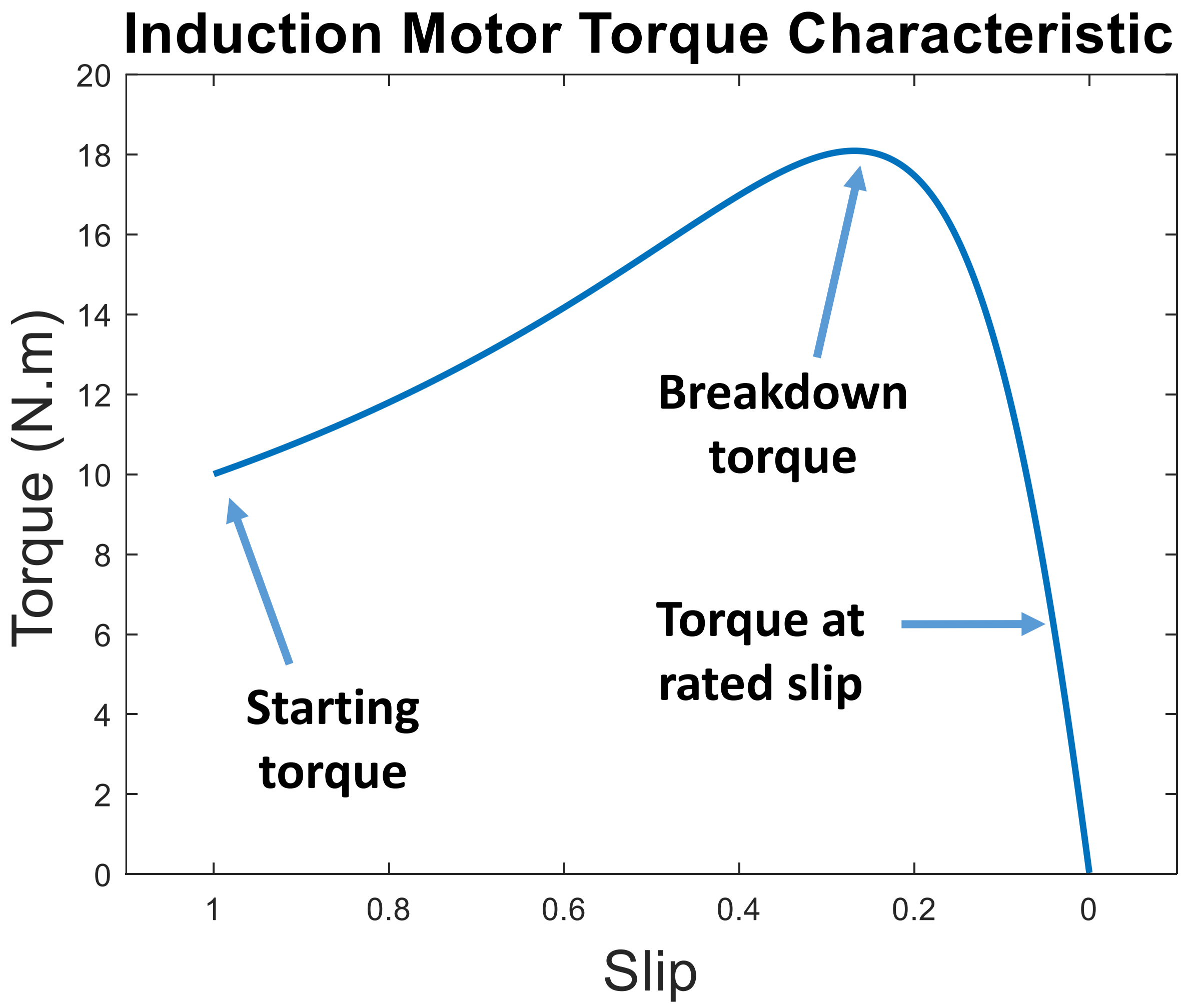
$$n_s = \frac{120f}{P} = \frac{120(60\text{Hz})}{4} = 1800\text{rpm}$$

$$\omega_s = \frac{n_s}{9.55}$$

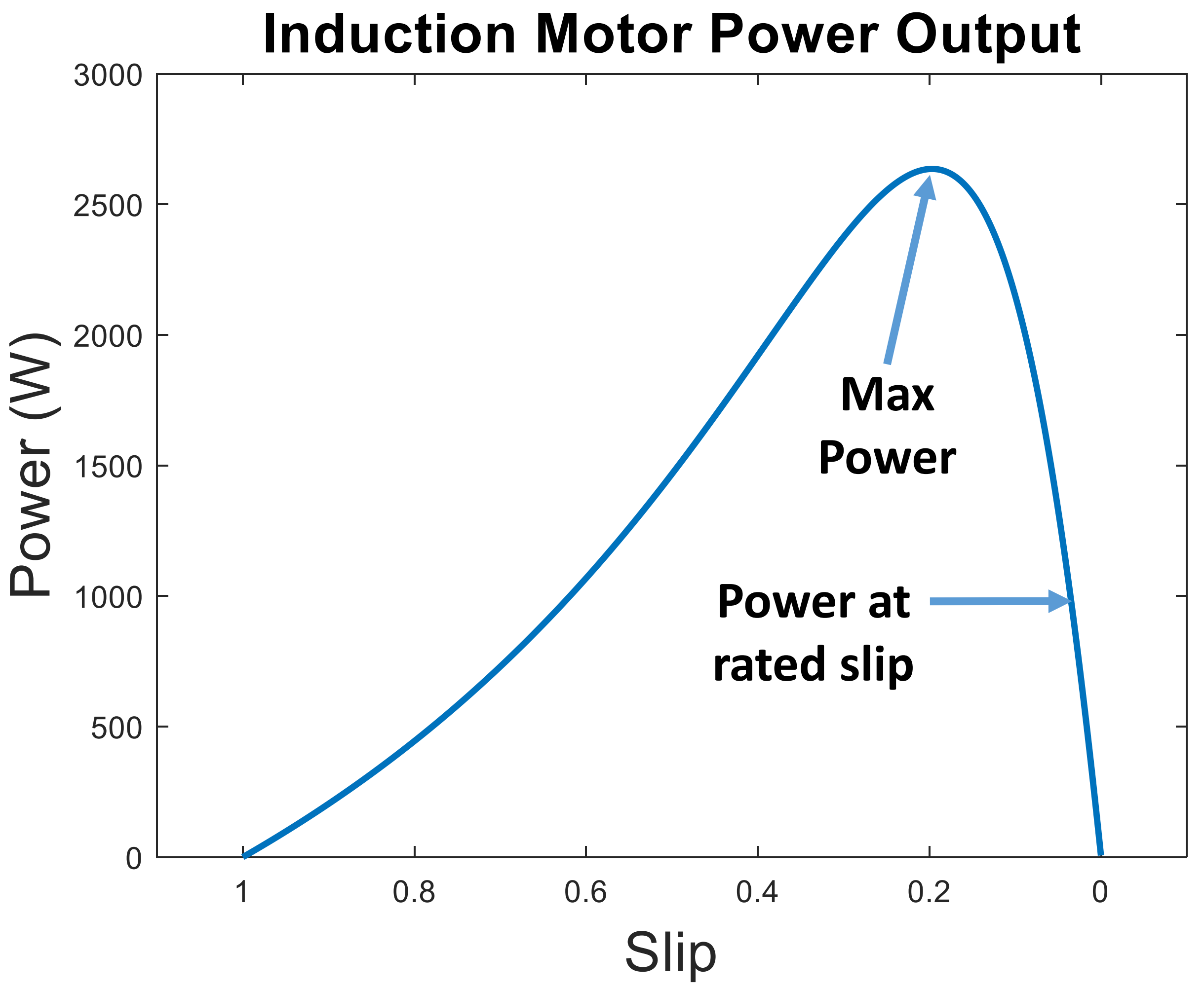
$$= 188.5 \text{ rad/s}$$



Induction Motor Example

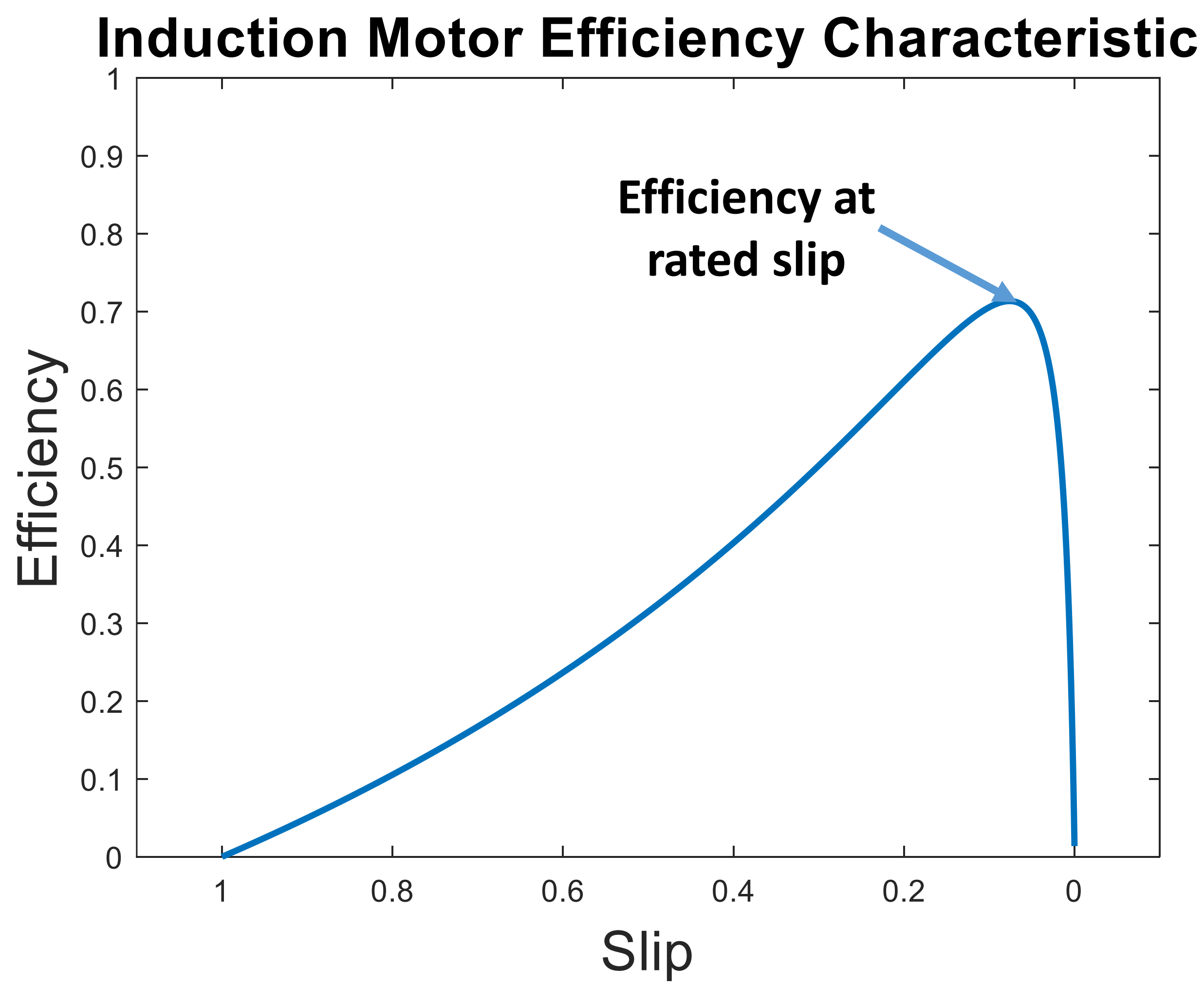


Induction Motor Example



Why is the rated slip not set for maximum power?

Induction Motor Example



Rated slip corresponds to optimal efficiency